

MARKING SCHEME (PRACTICE PAPER- 2)

PART – A SECTION – I

1. $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$ OR 1
2. $R = R^{-1}$
3. 2 OR 60
4. 2, 4
5. I OR Null Matrix
6. 32
7. $e^x \sec x + C$ OR $\tan x - \sec x + C$
8. 2 sq units
9. $\cot x$ OR 3
10. $4\hat{i} - 2\hat{j} + 4\hat{k}$
11. $5/2$
12. $\sqrt{2}$
13. $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$
14. $\sqrt{14}$
15. 0.75
16. $1/8$

SECTION II

17. (i) B (ii) C (iii) D (iv) A (v) B
18. (i) D (ii) C (iii) B (iv) D (v) A

PART – B SECTION III

19. Given that, $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$ 2

$$\Rightarrow \int e^{2y} dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C$$

When $x = 5$ and $y = 0$, then substituting these values in Eq. (i), we get

$$\frac{e^0}{2} = 5 + C$$

$$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$$

Eq. (i) becomes $e^{2y} = 2x - 9$

When $y = 3$, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$$\therefore x = \frac{(e^6 + 9)}{2}.$$

20. The equation of the line passing through $A(-1, 3, 2)$ and $B(-4, 2, -2)$ is

2

$$\frac{x+1}{-4+1} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$$

$$\Rightarrow \frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$$

$$\Rightarrow \frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4} \quad \dots (i)$$

If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, then the coordinates of C must satisfy equation (i). Therefore,

$$\frac{5+1}{3} = \frac{5-3}{1} = \frac{\lambda-2}{4}$$

$$\Rightarrow \frac{\lambda-2}{4} = 2$$

$$\Rightarrow \lambda = 10.$$

21.

Firstly, we find a unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

2

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$$

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$$\begin{aligned} \text{Now, vector of magnitude 6 units} &= 6 \left[\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k} \right] \\ &= 4\hat{i} - 2\hat{j} + 4\hat{k} \end{aligned}$$

22. We know that the principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cot^{-1}x$ is $(0, \pi)$.

2

$$\therefore \text{Principal value of } \tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right)$$

$$= \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\pi + \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}.$$

23.

$$I = \int_0^{\frac{\pi}{2}} \log \left(\frac{3 + 5 \cos \left(\frac{\pi}{2} - x \right)}{3 + 5 \sin \left(\frac{\pi}{2} - x \right)} \right) dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \log \left(\frac{3 + 5 \sin x}{3 + 5 \cos x} \right) dx = -I$$

$$2I = 0 \Rightarrow I = 0$$

2

OR The differentiation of $\log(\sin x)$ is $\cot x$, which exists in denominator. So solve by substitution method.

2

Given integral is $\int \frac{\log(\sin x)}{\tan x} dx$

Putting $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x dx = dt$$

$$\Rightarrow \cot x dx = dt$$

$$\Rightarrow \frac{1}{\tan x} dx = dt \quad \left[\because \int \sin x dx = -\cos x + C \right]$$

\therefore We get $\int \frac{\log(\sin x)}{\tan x} dx = \int t dt$

$$= \frac{t^2}{2} + C$$

$$= \frac{(\log \sin x)^2}{2} + C$$

24. We have,

$$x = a \cos^3 \theta, y = a \sin^3 \theta$$

2

$$\Rightarrow \frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta$$

and $\frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$$\Rightarrow \frac{dy}{dx} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta} = -\tan \theta$$

$$\therefore \text{Slope of the normal at any point on the curve} = -\frac{1}{\frac{dy}{dx}} = \frac{-1}{-\tan \theta} = \cot \theta$$

Hence, $\left(\text{Slope of the normal at } \theta = \frac{\pi}{4} \right) = \cot \frac{\pi}{4} = 1.$

25.

2

$$\begin{aligned}
 y &= \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) \\
 &= \tan^{-1} \left\{ \frac{\sin \left(\frac{\pi}{2} + x \right)}{1 - \cos \left(\frac{\pi}{2} + x \right)} \right\} \\
 &= \tan^{-1} \left\{ \frac{2 \sin \left(\frac{\pi}{4} + \frac{x}{2} \right) \cos \left(\frac{\pi}{4} + \frac{x}{2} \right)}{2 \sin^2 \left(\frac{\pi}{4} + \frac{x}{2} \right)} \right\} \\
 &= \tan^{-1} \left\{ \cot \left(\frac{\pi}{4} + \frac{x}{2} \right) \right\} \\
 &= \tan^{-1} \left\{ \tan \frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{x}{2} \right) \right\} \\
 &= \tan^{-1} \left\{ \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right\} \\
 &= \frac{\pi}{4} - \frac{x}{2}
 \end{aligned}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2}.$$

26.

2

We have, $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow (X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$\text{and, } (X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$Y = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

OR Let $A(5, 5)$, $B(k, 1)$ and $C(11, 7)$ be three vertices of a ΔABC .

2

Then area of $\Delta ABC = \frac{1}{2} \begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$

$$= \frac{1}{2} [5(1-7) - 5(k-11) + 1(7k-11)]$$

$$= \frac{1}{2} [-30 - 5k + 55 + 7k - 11]$$

$$= \frac{1}{2} [2k + 14] = (k + 7) \text{ sq. units}$$

But it is given that $A(5, 5)$, $B(k, 1)$ and $C(11, 7)$ are collinear.

$$\therefore \text{area of } \Delta ABC = 0$$

$$\therefore k + 7 = 0 \Rightarrow k = -7$$

27.

Let A and B be the events of drawings an even number ticket in the first and second drawn respectively.

2

In the first draw, there are 7 even numbers out of 15 numbers.

$$\therefore P(A) = \frac{7}{15}$$

After first draw, there are 14 tickets left.

In the second drawn, one even number ticket is drawn out of 14 tickets.

$$\therefore P(B|A) = \frac{6}{14} = \frac{3}{7}$$

$$\therefore P(A \cap B) = P(A) \cdot P(B|A) = \frac{7}{15} \times \frac{3}{7} = \frac{1}{5}$$

OR

When a die is thrown, there are 3 odd numbers on the die out of 6 numbers.

2

$$\therefore \text{Probability of getting odd number} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Probability of getting even number} = 1 - \frac{1}{2} = \frac{1}{2}$$

Now probability of getting no odd number when the die is tossed thrice

= Probability of getting even number when the die is tossed thrice

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

\therefore Probability of getting an odd number at least once when the die is tossed thrice

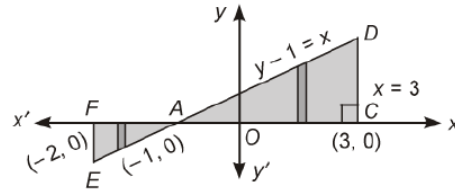
$$= 1 - \frac{1}{8} = \frac{7}{8}$$

28.

 $y - 1 = x$ or $y = x + 1$ is the given line DE .

... (i)

2

 $x = -2$ is the line EF . $x = 3$ is the line CD .Let A be a point of intersection of (i) and x -axis.Limits are $x = -2$ and $x = -1$ for the area AEF and the limits for the area ACD are $x = -1$ and $x = 3$.

The required area = Shaded area

$$= |\Delta AFE| + |\Delta ACD|$$

$$= \left| \int_{-2}^{-1} (x+1) dx \right| + \left| \int_{-1}^3 (x+1) dx \right|$$

$$= \left| \left[\frac{x^2}{2} + x \right]_{-2}^{-1} \right| + \left| \left[\frac{x^2}{2} + x \right]_{-1}^3 \right|$$

$$= \left| \left(\frac{1}{2} - 1 \right) - (2 - 2) \right| + \left| \left(\frac{9}{2} + 3 \right) - \left(\frac{1}{2} - 1 \right) \right|$$

SECTION IV

29.

 $R = \{(a, b) : a - b \text{ is divisible by } 5, a, b \in \mathbb{Z}\}$

3

For reflexive: $(a, a) \in R \Rightarrow a - a$ is divisible by 5, true. Hence R is reflexive.**For symmetric:** $(a, b) \in R \Rightarrow a - b$ divisible by 5 $\Rightarrow b - a$ is divisible by 5 $\Rightarrow (b, a) \in R$, Hence R is symmetric.**For transitive:** Let for $(a, b), (b, c) \in R$

$$(a, b) \in R \Rightarrow a - b \text{ divisible by } 5$$

$$(b, c) \in R \Rightarrow b - c \text{ divisible by } 5$$

As $a - b$ divisible by 5 and $b - c$ divisible by 5. Hence $a - c$ is also divisible by 5

$$\text{i.e., } (a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R. \text{ Hence } R \text{ is transitive.}$$

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

30.

Given curve is

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$$y = 1 + |x + 1| = \begin{cases} 1 + x + 1, & \text{if } x + 1 \geq 0 \\ 1 - (x + 1), & \text{if } x + 1 < 0 \end{cases}$$

or,

$$y = \begin{cases} x + 2, & \text{if } x \geq -1 \\ -x, & \text{if } x < -1 \end{cases} \quad \dots \text{ (i)}$$

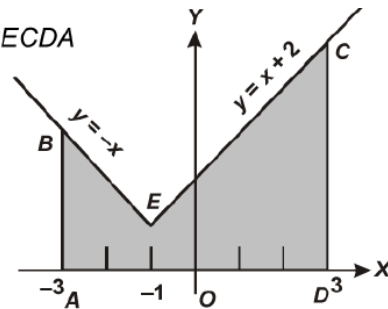
Given lines are

$$x = -3, \quad x = 3, \quad y = 0 \quad \dots \text{ (ii)}$$

The rough sketch of (i) has been shown in the figure.

∴ The required area = the area of the shaded region $ABECDA$

$$\begin{aligned}
 &= \int_{-3}^{-1} y_{BE} dx + \int_{-1}^3 y_{EC} dx \\
 &= \int_{-3}^{-1} (-x) dx + \int_{-1}^3 (x+2) dx \\
 &= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^3
 \end{aligned}$$



$$\begin{aligned}
 &= -\frac{1}{2}(1-9) + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} - 2\right)\right] \\
 &= 4 + 12 = 16 \text{ sq. units.}
 \end{aligned}$$

OR Given curves are $y^2 = 4x$... (i) 3
and $x + y = 3$... (ii)

Curve (i) is a right handed parabola whose vertex is (0, 0) and axis is $y = 0$.

Line (ii) cuts x -axis at (3, 0) and y -axis at (0, 3). Here required area $OCDAO$ is bounded by curves (i) and (ii) and abscissa at A and C.

Hence we will find the values of y from equations (i) and (ii).

Putting the value of x from equation (ii) in (i), we get

$$y^2 = 4(3 - y)$$

or, $y^2 + 4y - 12 = 0$

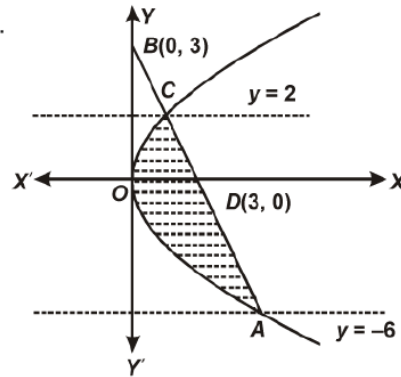
∴ $y = -6, 2$

Required area $OCDAO = \int_{-6}^2 (x_1 - x_2) dy$

$$= \int_{-6}^2 (x_{line(i)} - x_{curve(ii)}) dy$$

$$= \int_{-6}^2 \left[(3 - y) - \frac{y^2}{4} \right] dy = \left[3y - \frac{y^2}{2} - \frac{y^3}{12} \right]_{-6}^2$$

$$= \left(6 - 2 - \frac{2}{3} \right) - \left(-18 - 18 + \frac{216}{12} \right) = \frac{10}{3} + 18 = \frac{64}{3} \text{ sq. units.}$$



31. Let $I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$... (i) 3

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a - x) dx \right]$$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \quad \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{(x + \pi - x) \sin x}{1 + \cos^2 x} dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx \quad [\because \text{Let } t = \cos x, dt = -\sin x dx]$$

$$\left[\begin{array}{l} \text{upper limit} \rightarrow \cos \pi = -1 \\ \text{Lower limit} \rightarrow \cos 0 = 1 \end{array} \right]$$

$$I = \frac{\pi}{2} \int_1^{-1} \frac{-dt}{1+t^2} = \frac{\pi}{2} \int_{-1}^1 \frac{dt}{1+t^2} \quad \left[\because \int_a^b f(x) dx = - \int_b^a f(x) dx \right]$$

$$= \frac{\pi}{2} [\tan^{-1} t]_{-1}^1 = \frac{\pi}{2} [\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$= \frac{\pi}{2} \left[\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right] = \frac{\pi}{2} \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = \frac{\pi}{2} \cdot \frac{\pi}{2}$$

$$= \frac{\pi^2}{4}$$

32. We have,

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$$f(x) = (x+1)^3 (x-3)^3$$

$$\Rightarrow f'(x) = (x-3)^3 \cdot 3(x+1)^2 \frac{d}{dx}(x+1) + (x+1)^3 \cdot 3(x-3)^2 \frac{d}{dx}(x-3)$$

$$\Rightarrow f'(x) = 3(x+1)^2 (x-3)^3 + 3(x+1)^3 (x-3)^2$$

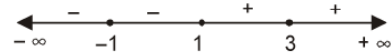
$$\Rightarrow f'(x) = 3(x+1)^2 (x-3)^2 (x+1+x-3)$$

$$\Rightarrow f'(x) = 6(x+1)^2 (x-3)^2 (x-1)$$

For $f(x)$ to be increasing, we must have

$$f'(x) \geq 0$$

$$\Rightarrow 6(x+1)^2 (x-3)^2 (x-1) \geq 0$$



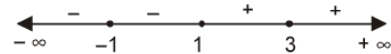
$$\Rightarrow x \in [1, \infty)$$

So, $f(x)$ is increasing on $[1, \infty)$.

For $f(x)$ to be decreasing, we must have

$$f'(x) \leq 0$$

$$\Rightarrow 6(x+1)^2 (x-3)^2 (x-1) \leq 0$$



$$\Rightarrow x \in (-\infty, 1]$$

So, $f(x)$ is decreasing on $(-\infty, 1]$.

33. Given that

3

$$x = \tan\left(\frac{1}{a} \log y\right)$$

$$\Rightarrow \tan^{-1} x = \frac{1}{a} \log y$$

$$\Rightarrow a \tan^{-1} x = \log y$$

Now, differentiating both sides w.r.t. x , we get

$$a \times \frac{1}{1+x^2} = \frac{1}{y} \cdot \frac{dy}{dx} \quad \left[\because \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} \right]$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = ay \quad \text{[By cross multiplication]}$$

Differentiating again on both sides w.r.t. x , we get

$$(1+x^2) \cdot \frac{d}{dx} \left(\frac{dy}{dx} \right) + \frac{dy}{dx} \frac{d}{dx} (1+x^2) = \frac{d}{dx} (ay) \quad \left[\because \frac{d}{dx} (u \cdot v) = u \frac{dv}{dx} + v \frac{du}{dx} \right]$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (2x) = a \cdot \frac{dy}{dx}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - a \frac{dy}{dx} = 0$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + (2x-a) \frac{dy}{dx} = 0. \quad \text{Hence proved.}$$

34.

$$x = ae^\theta (\sin \theta - \cos \theta)$$

3

Diff. w.r.t. θ

$$\frac{dx}{d\theta} = a[e^\theta (\cos \theta + \sin \theta) + (\sin \theta - \cos \theta) e^\theta]$$

$$= ae^\theta [\cos \theta + \sin \theta + \sin \theta - \cos \theta]$$

$$= 2ae^\theta \sin \theta$$

Now,

$$y = ae^\theta (\sin \theta + \cos \theta)$$

Diff. w.r.t. θ

$$\frac{dy}{d\theta} = a[e^\theta (\cos \theta - \sin \theta) + (\sin \theta + \cos \theta) e^\theta]$$

$$= ae^\theta [\cos \theta - \sin \theta + \sin \theta + \cos \theta]$$

$$= 2ae^\theta \cos \theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2ae^\theta \cos \theta}{2ae^\theta \sin \theta}$$

$$= \cot \theta.$$

OR

Given that

$$y^x = e^{y-x}$$

3

Taking log on both sides, we get

$$\log y^x = \log e^{(y-x)}$$

$$\Rightarrow x \log y = (y-x) \log e \quad [\because \log e = 1]$$

$$\Rightarrow x \log y = y - x \quad \dots (i)$$

$$\Rightarrow x = \frac{y}{1 + \log y}$$

Differentiating eq. (i) both sides w.r.t. x , we get

$$\Rightarrow x \cdot \frac{d}{dx} (\log y) + \log y \frac{d}{dx} (x) = \frac{d}{dx} (y) - \frac{d}{dx} (x)$$

$$\Rightarrow x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1 = \frac{dy}{dx} - 1$$

$$\Rightarrow (1 + \log y) = \frac{dy}{dx} \left(1 - \frac{x}{y}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(1 + \log y)}{(y - x)} \quad \dots \text{(ii)}$$

Put the value of x from Eq. (i) in Eq. (ii), we get

$$\frac{dy}{dx} = \frac{y(1 + \log y)}{y - \left(\frac{y}{1 + \log y}\right)} \quad \left[\begin{array}{l} \because x \log y = y - x \\ \text{or } x = \frac{y}{1 + \log y} \end{array} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(1 + \log y)^2}{(y + y \log y - y)}$$

35. Given differential equation is

3

$$(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}; |x| \neq 1$$

Dividing both sides by $x^2 - 1$, we get

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = \frac{1}{(x^2 - 1)^2} \quad \dots \text{(i)}$$

This is a linear differential equation and is of the form

$$\frac{dy}{dx} + Py = Q \quad \dots \text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$P = \frac{2x}{x^2 - 1} \text{ and } Q = \frac{1}{(x^2 - 1)^2}$$

Now, solution of above equation is given by

$$y \times \text{I.F.} = \int (Q \times \text{I.F.}) dx + c \quad \dots \text{(iii)}$$

where, I.F. = Integrating factor and I.F. = $e^{\int P dx}$

$$\therefore \text{I.F.} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = x^2 - 1 \quad [\because e^{\log x} = x]$$

$$\left[\because \int \frac{2x}{x^2 - 1} dx \text{ Put } x^2 - 1 = t \Rightarrow 2x dx = dt \therefore \int \frac{dt}{t} = \log |t| = \log |x^2 - 1| + c \right]$$

Putting I.F. = $x^2 - 1$ and $Q = \frac{1}{(x^2 - 1)^2}$ in Eq. (iii),

we get

$$y(x^2 - 1) = \int (x^2 - 1) \cdot \frac{1}{(x^2 - 1)^2} dx$$

$$\Rightarrow y(x^2 - 1) = \int \frac{1}{x^2 - 1} dx$$

$$\Rightarrow y(x^2 - 1) = \int \frac{dx}{x^2 - (1)^2}$$

$$y(x^2 - 1) = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$$

SECTION – V

36. Let $P(2, 3, 4)$ be the given point and given equation of line be

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Any random point T on the given lines is calculated as

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2} = \lambda \quad [\text{Say}]$$

or $x = 3\lambda - 3, y = 6\lambda + 2, z = 2\lambda$

\therefore Coordinates of T are $(3\lambda - 3, 6\lambda + 2, 2\lambda)$

Now, DR's of line PT are

$$(3\lambda - 3 - 2, 6\lambda + 2 - 3, 2\lambda - 4) = (3\lambda - 5, 6\lambda - 1, 2\lambda - 4)$$

Since, the line PT is parallel to the plane

$$3x + 2y + 2z - 5 = 0$$

$$\therefore a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

[\because Line is parallel to the plane, therefore normal to the
plane is perpendicular to the line]

where, $a_1 = 3\lambda - 5, b_1 = 6\lambda - 1, c_1 = 2\lambda - 4$

and $a_2 = 3, b_2 = 2, c_2 = 2$

[$\because a_2, b_2, c_2$ are DR's of plane whose equation
is $3x + 2y + 2z - 5 = 0$]

\therefore We get,

$$3(3\lambda - 5) + 2(6\lambda - 1) + 2(2\lambda - 4) = 0$$

$$\Rightarrow 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$$

$$\Rightarrow 25\lambda - 25 = 0$$

$$\Rightarrow 25\lambda = 25$$

or $\lambda = 1$

\therefore Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda) = (0, 8, 2)$ [Put $\lambda = 1$]

Finally, the required distance between points $P(2, 3, 4)$ and $T(0, 8, 2)$ is given by

$$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$$

$$[\because (x_1, y_1, z_1) = (2, 3, 4) \text{ and } (x_2, y_2, z_2) = (0, 8, 2)]$$

$$= \sqrt{4 + 25 + 4} = \sqrt{33} \text{ units.}$$

OR We have, $\vec{n}_1 = (\hat{i} + 3\hat{j})$, $d_1 = 6$ and $\vec{n}_2 = (3\hat{i} - \hat{j} - 4\hat{k})$, $d_2 = 0$

Using the relation, $\vec{r} \cdot (\vec{n}_1 + \lambda\vec{n}_2) = d_1 + d_2\lambda$

$$\Rightarrow \vec{r} \cdot [(\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} - 4\hat{k})] = 6 + 0 \cdot \lambda$$

$$\Rightarrow \vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)] = 6 \quad \dots (i)$$

On dividing both sides by $\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}$, we get

$$\frac{\vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)]}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} = \frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}}$$

Since, the perpendicular distance from origin is unity.

$$\therefore \frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} = 1$$

$$\Rightarrow (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2 = 36$$

$$\Rightarrow 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2 = 36$$

$$\Rightarrow 26\lambda^2 + 10 = 36$$

$$\Rightarrow \lambda^2 = 1$$

$$\therefore \lambda = \pm 1$$

Using Eq. (i), the required equation of plane is

$$\vec{r} \cdot [(1 \pm 3)\hat{i} + (3 \mp 1)\hat{j} + (\mp 4)\hat{k}] = 6$$

$$\Rightarrow \vec{r} \cdot [(1 + 3)\hat{i} + (3 - 1)\hat{j} + (-4)\hat{k}] = 6$$

and $\vec{r} \cdot [(1 - 3)\hat{i} + (3 + 1)\hat{j} + 4\hat{k}] = 6$

$$\Rightarrow \vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) = 6$$

and $\vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) = 6$

$$\Rightarrow 4x + 2y - 4z - 6 = 0$$

and $-2x + 4y + 4z - 6 = 0$

37. Maximise $z = 1000x + 600y$
 Subject to $x + y \leq 200, x \geq 20, y \geq 4x, x, y \geq 0$.

Consider the linear constraint defined by the inequality

$$x + y \leq 200$$

First draw the graph of the line $x + y = 200$

x	100	80
y	100	120

Putting $(0, 0)$ in the inequality $x + y \leq 200$, we have

$$0 + 0 \leq 200 \Rightarrow 0 \leq 200, \text{ which is true}$$

So the half plane of $x + y \leq 200$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x \geq 20$$

First draw the graph of the line $x = 20$

Putting $(0, 0)$ in the inequality $x \geq 20$, we have

$$0 \geq 20, \text{ which is false}$$

So the half plane of $x \geq 20$ is away from origin.

Now consider the linear constraint defined by the inequality

$$y \geq 4x$$

First draw the graph of the line $y = 4x$

x	10	20
y	40	80

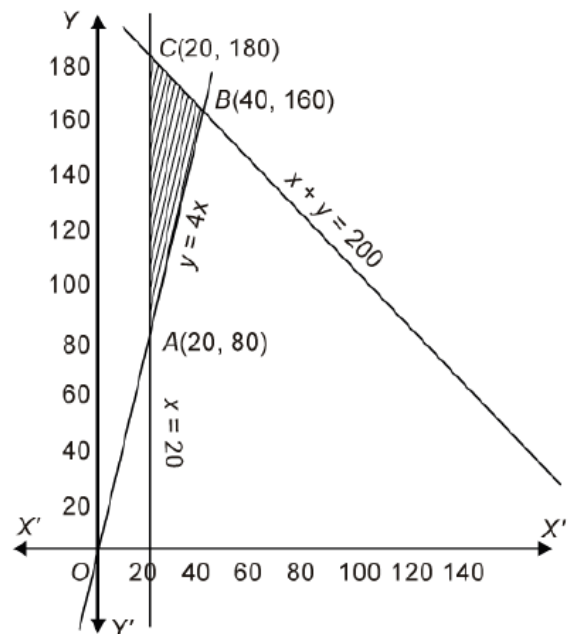
Putting $(10, 0)$ in the inequality $y \geq 4x$, we have

$$0 \geq 4 \times 10 \Rightarrow 0 \geq 40, \text{ which is false}$$

So the half plane of $y \geq 4x$ is away from the point $(10, 0)$.

Since $x, y \geq 0$

So the feasible region lies in the first quadrant.



The coordinates of the corner points of the feasible region are $A(20, 80)$, $B(40, 160)$ and $C(20, 180)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 1000x + 600y$

At $A(20, 80)$ $z = 1000 \times 20 + 600 \times 80 = 20000 + 48000 = 68000$

At $B(40, 160)$ $z = 1000 \times 40 + 600 \times 160 = 40000 + 96000 = 136000$

At $C(20, 180)$ $z = 1000 \times 20 + 600 \times 180 = 20000 + 108000 = 128000$

Thus z is maximum at $(40, 160)$ and maximum value = 136000

OR.

The given objective function is $z = 6x + 5y$

Consider the linear constraint defined by the inequality

$$3x + 5y \leq 15$$

First draw the graph of the line $3x + 5y = 15$

x	0	5
y	3	0

Putting $(0, 0)$ in the inequality $3x + 5y \leq 15$, we have

$$3 \times 0 + 5 \times 0 \leq 15 \Rightarrow 0 \leq 15, \text{ which is true}$$

So the half plane of $3x + 5y \leq 15$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$5x + 2y \leq 10$$

First draw the graph of the line $5x + 2y = 10$

x	0	2
y	5	0

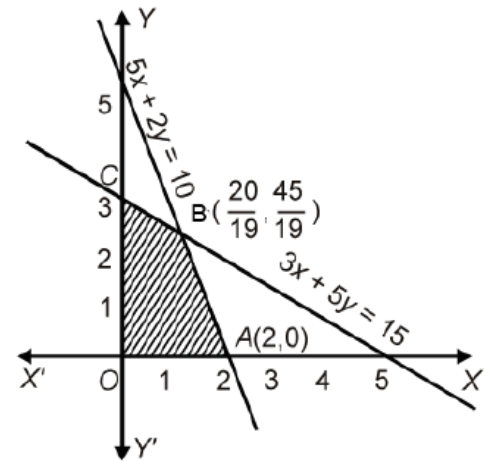
Putting $(0, 0)$ in the inequality $5x + 2y \leq 10$, we have

$$5 \times 0 + 2 \times 0 \leq 10 \Rightarrow 0 \leq 10, \text{ which is true}$$

So the half plane of $5x + 2y \leq 10$ is towards the origin.

Since $x, y \geq 0$

So the feasible region lies in first quadrant.



The coordinates of the corner points of the feasible region are $O(0, 0)$

$A(2, 0)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$. These points have been obtained by solving equations of the

corresponding intersecting lines simultaneously.

Now $z = 6x + 5y$

At $O(0, 0)$ $z = 6 \times 0 + 5 \times 0 = 0$

At $A(2, 0)$ $z = 6 \times 2 + 5 \times 0 = 12 + 0 = 12$

At $B\left(\frac{20}{19}, \frac{45}{19}\right)$ $z = 6 \times \frac{20}{19} + 5 \times \frac{45}{19} = \frac{120}{19} + \frac{225}{19} = \frac{345}{19}$

At $C(0, 3)$ $z = 6 \times 0 + 5 \times 3 = 0 + 15 = 15$

Thus z is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and maximum value = $\frac{345}{19}$.

38. Given that

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$$

The given system of equations can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

Solution of above system of equations is given by

$$X = A^{-1}B \quad \dots (i)$$

So, now we find A^{-1} ,

where $A^{-1} = \frac{\text{adj}(A)}{|A|}$

Now, $|A| = 3(3 - 6) - 2(-12 - 14) + 1(12 + 7)$

$$= 3(-3) - 2(-26) + 1(19)$$

$$= -9 + 52 + 19 = 62$$

$$|A| \neq 0,$$

hence unique solution.

and

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

\therefore

$$A_{11} = (-1)^2 \begin{vmatrix} -1 & 2 \\ 3 & -3 \end{vmatrix} = (-1)^2 \times (3 - 6) = -3$$

$$A_{12} = (-1)^3 \begin{vmatrix} 4 & 2 \\ 7 & -3 \end{vmatrix} = -1(-12 - 14) = 26$$

$$A_{13} = (-1)^4 \begin{vmatrix} 4 & -1 \\ 7 & 3 \end{vmatrix} = 1(12 + 7) = 19$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} = -1(-6 - 3) = 9$$

$$A_{22} = (-1)^4 \begin{vmatrix} 3 & 1 \\ 7 & -3 \end{vmatrix} = 1(-9 - 7) = -16$$

$$A_{23} = (-1)^5 \begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = -1(9 - 14) = 5$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 1(4 + 1) = 5$$

$$A_{32} = (-1)^5 \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = -1(6 - 4) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = 1(-3 - 8) = -11$$

\therefore

$$\text{adj}(A) = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

\Rightarrow

$$A^{-1} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$\left[\because A^{-1} = \frac{\text{adj}(A)}{|A|} \right]$$

Now, by using Eq. (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$= \frac{1}{62} \begin{bmatrix} -18 + 45 + 35 \\ 156 - 80 - 14 \\ 114 + 25 - 77 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$$\therefore x = \frac{62}{62}; \quad y = \frac{62}{62}; \quad z = \frac{62}{62}$$

Hence, $x = 1$, $y = 1$ and $z = 1$.

OR

First find the product AB and then premultiply both sides of product AB by A^{-1} and obtain A^{-1} . Then, using the relation $X = A^{-1}C$ and simplify it to get the result.

First we find the product AB

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$$

$$\therefore AB = 6I \quad \dots(i)$$

Now, given system of equations can be written as

$$AX = C \Rightarrow X = A^{-1}C \quad \dots (ii)$$

where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

Now, again from Eq. (i)

$$AB = 6I$$

$$\Rightarrow A^{-1}AB = 6A^{-1}I$$

[Premultiplying by A^{-1} on both sides]

$$\Rightarrow B = 6A^{-1}$$

[$\because A^{-1}A = I$ and $IB = B$]

$$\therefore A^{-1} = \frac{1}{6}B = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now from Eq. (ii), we get

$$X = A^{-1}C \quad \text{where} \quad C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\therefore x = 2, \quad y = -1 \quad \text{and} \quad z = 4.$$